ELECTRIC FIELDS IN MAGNETOHYDRODYNAMIC CHANNELS IN THE PRESENCE OF ELECTRODE POTENTIAL DROP

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It is well known that under certain conditions of operation of magnetohydrodynamic machines a significant influence is exerted by the processes taking place in the electrode layers, which cannot be described by the usual equations of magnetohydrodynamics. In spite of the microscopic thickness of the electrode layer regions, the electric potential undergoes a finite change in them, sometimes amounting to a significant fraction of the induced or imposed potential differences. Accordingly, the results of calculation of electric fields in channels without taking into account the electrode layer effects* can, in certain cases, lead to incorrect prediction of the performance of magnetohydrodynamic machines. A very complete theoretical study of electrode layer process was carried out by Liubimov [2 to 6]. On the basis of these papers, and of other experimental work, the drop of potential $\delta\varphi^\circ$ in the electrode layer can, in a number of cases, be successfully related to the characteristics of the electrode and the gasdynamic and electrical parameters on the surface of the electrode layer. These results can be used for studying the influence of the electrode layer processes on three-dimensional effects in channels. The present paper is devoted to the formulation of the relevant problems and to the solution of some of them.

1. As follows from the theory of the electrode layer, its thickness is negligibly small in comparison with the dimensions of the system (in many cases it is less than the length of the path of charged particles). This permits the conditions on the surface of the electrode layer to be satisfied at the surface of the electrode and allows us to assume that in passing through the layer the normal component of electric current density j_n° is conserved. We shall moreover assume that the function $\delta\varphi^\circ = f(j_n^\circ, \ldots)$, defining the dependence of the electrode layer potential drop on the other parameters, is known either from experiment, or from

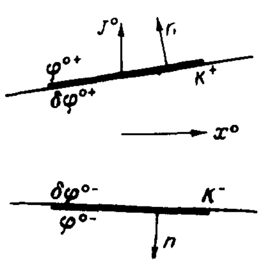

FIG. 1

* An approximate method of making such a calculation is describes in ref. [1].

theoretical consideration of the electrode layer. It will be assumed that this function does not depend explicitly upon the strength of the magnetic field $\mathbf{B}^\circ$. The precise knowledge of this dependence together with the usual boundary conditions for the electrical quantities ($j_n{}^\circ = 0$ on the nonconducting segments of the walls of the channel, φ° = const along the electrode) and the appropriate boundary conditions for the gasdynamic quantities, makes it possible in a number of cases to complete the system of the magnetogasdynamic equations for the region outside the electrode layers.

Let us suppose that the walls of the channel are nonconducting, except for the segments K^+ and K^-, the distribution of potential and the electrode layer drop of potential along which, are given by the functions $\varphi^{\circ+}$, $\varphi^{\circ-}$ and $\delta\varphi^{\circ+}$, $\delta\varphi^{\circ-}$ respectively (fig. 1.) Assuming the process to be steady, and the current density when $|x^\circ| \to \infty$ to be equal to zero, and using the assumption that the thickness of the electrode layer is small, we obtain the equation.

$$\iiint \mathbf{j}^\circ \mathbf{E}^\circ dD = -\iiint \operatorname{div} \varphi^\circ \mathbf{j}^\circ dD =$$

$$= -\iint_{K+} (\varphi^{\circ+} + \delta\varphi^{\circ+})\, j_n{}^\circ d\Sigma - \iint_{K-} (\varphi^{\circ-} + \delta\varphi^{\circ-})\, j_n{}^\circ d\Sigma \qquad (\mathbf{E}^\circ = -\nabla\varphi^\circ) \tag{1.1}$$

Here the volume integral extends over the region of the channel outside the electrode layers, the double intergrals are taken over the surfaces of the segments K^+ and K^-, $\mathbf{E}^\circ$ is the electric field vector and $\mathbf{n}$ is the outer normal. On the other hand we have

$$\mathbf{j}^\circ = \sigma\left(\mathbf{E}^\circ + \frac{1}{c}\mathbf{v}^\circ \times \mathbf{B}^\circ\right) \tag{1.2}$$

$$\iiint \mathbf{j}^\circ \mathbf{E}^\circ dD = \iiint \mathbf{j}^\circ\left(\frac{\mathbf{j}^\circ}{\sigma} - \frac{1}{c}\,\mathbf{v}^\circ \times \mathbf{B}^\circ\right) dD = Q - A$$

$$\left(Q = \iiint \frac{\mathbf{j}^{\circ 2}}{\sigma} dD,\ A = -\frac{1}{c}\iiint \mathbf{v}^\circ (\mathbf{j}^\circ \times \mathbf{B}^\circ)\, dD\right) \tag{1.3}$$

Equation (1.2), where σ is the electrical conductivity, $\mathbf{v}^\circ$ is the velocity vector and c is the velocity of light *in vacuo*, is the simplest expression of Ohm's Law, Q and A are the Joule dissipation and the work of the medium (per unit time) in overcoming the resistance of the magnetic field, respectively. Combining (1.1) and (1.3) we find that

$$A = Q + \iint_{K+} \varphi^{\circ+} j_n{}^\circ d\Sigma + \iint_{K-} \varphi^{\circ-} j_n{}^\circ d\Sigma + \iint_{K+} \delta\varphi^{\circ+} j_n{}^\circ d\Sigma + \iint_{K-} \delta\varphi^{\circ-} j_n{}^\circ d\Sigma \tag{1.4}$$

If K^+ and K^- are connected, then

$$\iint_{K+} \varphi^{\circ+} j_n{}^\circ d\Sigma + \iint_{K-} \varphi^{\circ-} j_n{}^\circ d\Sigma = J^\circ(\varphi^{\circ+} - \varphi^{\circ-}) \qquad \left(J^\circ = \iint_{K+} j_n{}^\circ d\Sigma\right) \tag{1.5}$$

In agreement with (1.4) in the generating mode the work A is converted into Joule dissipation, electrical energy used up on the external load, and the losses in the electrode layers. The efficiency of such an apparatus is determined from the formula

$$\eta^* = \frac{J^\circ(\varphi^{\circ+} - \varphi^{\circ-})}{A} \tag{1.6}$$

If the device is used as an accelerator, then the electrical energy supplied to the apparatus $-J^{\circ}(\varphi^{\circ+}-\varphi^{\circ-})$ is equal to the Joule dissipation, the losses in the electrode layers and the work $-A$ of the electromagnetic forces in accelerating the gas. The efficiency of the apparatus is therefore equal to

$$\eta^{**}=\frac{-A}{-J^{\circ}(\varphi^{\circ+}-\varphi^{\circ-})} \tag{1.7}$$

We note that the relation (1.4) remains valid even in presence of anisotropic conductivity in which case a vector collinear to $\mathbf{j}^{\circ}\times\mathbf{B}^{\circ}$ is added to the right-hand side of (1.2). The formulae (1.6) and (1.7) are easily generalised to the case of an arbitrary number of electrodes.

2. The complete solution of the problem of three-dimensional magnetogasdynamic flow in a channel in the majority of cases encounters formidable computational difficulties. However under certain conditions the calculation of the electric field in the channel can be carried out for specific values of the gasdynamic quantities [1]. In this approximation $\delta\varphi^{\circ}$ has to be taken as a function of the electrical parameters only. The boundary problem of the distribution of the current in the channel, with isotropic conductivity and small magnetic Reynolds numbers, is formulated in this approximation as follows:

$$\mathbf{j}^{\circ}=\sigma\Big(-\nabla\varphi^{\circ}+\frac{1}{c}\,\mathbf{v}^{\circ}\times\mathbf{B}^{\circ}\Big),\qquad \operatorname{div}\mathbf{j}^{\circ}=0$$

$$j_n^{\circ}=0 \text{ on the dielectrics},\quad \varphi^{\circ}=\varphi^{\circ\pm}+\delta\varphi^{\circ\pm}(j_n^{\circ}) \text{ on the electrodes} \tag{2.1}$$

$$\varphi^{\circ+}-\varphi^{\circ-}=J^{\circ}R$$

In (2.1) $\mathbf{j}^{0}$ and φ° are the unknowns and $\varphi^{\circ+}-\varphi^{\circ-}=\mathbf{j}^{0}R$. The last integral condition is used to determine the potential difference $\varphi^{\circ+}-\varphi^{\circ-}$ in the generating mode, across the external load R.

In a number of cases the electrode drop of potential is small in comparison with the applied or induced difference of potentials*, and the equations (2.1) can be linearised in the neighborhood of the solution with $\delta\varphi^{\circ}\equiv 0$. Assuming** that $\mathbf{B}^{0}=(0,0,-B^{0}(x^{0}))$, $\mathbf{v}^{\circ}=(0,0,V^{\circ}(x^{\circ}))$, $\sigma=\text{const}$, we obtain the equations in the zero and first approximation. Let us pass to dimensionless variables by means of the formulae

$$V^{\circ}=V^{*}V,\quad B^{\circ}=B^{*}B,\quad x^{\circ}=hx,\quad y^{\circ}=hy$$
$$\varphi^{\circ}=\frac{V^{*}B^{*}h}{c}\varphi,\quad \delta\varphi^{\circ}=\frac{V^{*}B^{*}h}{c}\delta\varphi,\quad \mathbf{j}^{\circ}=\frac{\sigma V^{*}B^{*}}{c}\mathbf{j} \tag{2.2}$$

Assuming $\delta\varphi=\varepsilon s(j_n)$, where $\varepsilon=\text{const}=o(1)$, $s=O(1)$, and representing the currents and the potential in form of series

$$\varphi=\varphi_0+\varepsilon\varphi_1+\ldots,\quad \varphi^{\pm}=\varphi_0^{\pm}+\varepsilon\varphi_1^{\pm}+\ldots,$$
$$j_x=j_{x0}+\varepsilon j_{x1}+\ldots,\ j_y=j_{y0}+\varepsilon j_{y1}+\ldots,\ s(j_n)=s(j_{n0})+\ldots \tag{2.3}$$

we find, from the system (2.1)

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* This condition for example is fulfilled for large scale channels when the flow of gas results in generation of electrical energy.

** [7 to 9] investigated the longitudinal boundary effect without the electrode layers for the values of σ, $\mathbf{B}^{0}$ and v^{0} given above.

$$j_{x0} = -\frac{\partial \varphi_0}{\partial x}, \qquad j_{y0} = -\frac{\partial \varphi_0}{\partial y} + f(x), \qquad \triangle \varphi_0 = 0 \qquad (f = VB)$$

$$j_{n0} = 0 \quad \text{on the dielectrics}, \qquad \varphi_0 = \varphi_0^{\pm} \quad \text{on the electrodes}$$

$$\varphi_0^{+} - \varphi_0^{-} = R\sigma J_0 \qquad \left(J_0 = \int_{K+} i_{n0}\, dl\right)$$

$$j_{x1} = -\frac{\partial \varphi_1}{\partial x}, \qquad j_{y1} = -\frac{\partial \varphi_1}{\partial y}, \qquad \triangle \varphi_1 = 0 \tag{2.4}$$

$$j_{n1} = 0 \quad \text{on the dielectrics}, \qquad \varphi_1 = \varphi_1^{\pm} + s^{\pm}(j_{n0}) \quad \text{on the electrodes}$$

$$\varphi_1^{+} - \varphi_1^{-} = R\sigma J_1 \qquad \left(J_1 = \int_{K+} i_{n1}\, dl\right) \tag{2.5}$$

In (2.2) the quantities V^*, B^* and h are the characteristic velocity, characteristic magnetic field intensity and characteristic linear size respectively and l is the dimensionless length along K^+. The system (2.4) defines the electric current without the electrode layer drop of potential and the first order corrections are found from system (2.5). The last relations in the systems (2.4) and (2.5) are used to determine the induced potential difference in the generator mode. If, on the other hand the potential difference is given, then, after the solution of the problem has been obtained, it can be used to determine the external load.

3. The quantity $\delta\varphi$ is determined from the volt-ampere characteristics obtained in the problem of current transmission between the electrodes*. In the general case the function $\delta\varphi\ (j_n)$ may be fairly complex, and that is inconvenient in obtaining the solution of system (2.1). Under certain conditions, however, $\delta\varphi$ can be represented by simple functions. According to [2 to 6] it can be assumed, that on the segment of the electrode with $j_n < 0$ (the electrons move from the stream into the electrode), $\delta\varphi = 0$ with a high degree of accuracy. If $j_n > 0$ and the surface of the electrode is a good ion absorber (e.g. graphite) then at low electrode temperatures $\delta\varphi \sim j_n$; if however the ions are reflected from the surface in any appreciable degree (e.g. off a tungsten electrode), then starting from a certain current density, $\delta\varphi = \text{const}$. For large currents the latter condition is probably true for most materials. The fact that the behaviour of the function $\delta\varphi$ along the surface of the electrode depends on the sign of j_n greatly complicates the solution of the system (2.1). Mathematically this is connected with the change of form of the boundary condition in passing through an unknown point of the boundary.

4. Let us derive the simplest solution of system (2.1). We shall consider a plane channel $|x^\circ| < \infty$, $|y^\circ| < {}^1/_2 h$ with electrodes placed symetrically $|x^\circ| < \lambda$, $|y^\circ| = \pm {}^1/_2\, h$. Let $\sigma = \text{const}$, $\mathbf{v}^\circ = (V^\circ(y^\circ), 0, 0)$, $\mathbf{B}^\circ = (0, 0, -B^*)$, $B^* = \text{const} > 0$. We shall assume that $\delta\varphi^\circ = \text{const} > 0$ and $\delta\varphi^\circ = 0$ on the electrodes with $j_n^\circ > 0$ and $j_n^\circ < 0$ respectively (fig. 2.) It is easy to see that the solution of system (2.1) is obtained from the solution [7] of the corresponding problem with $\delta\varphi^\circ \equiv 0$, if the electromotive force $\mathscr{E} = c^{-1}BG$, where G is the volume of the fluid expelled, is replaced with $\mathscr{E} - \delta\varphi^\circ$

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* In obtaining the volt-ampere characteristics a uniform distribution of the current between the electrodes should be aimed at. Then $\varphi^{\circ+} - \varphi^{\circ-} = -\delta\varphi^{\circ+} + \delta\varphi^{\circ-} + j^\circ r$, where r is the internal resistance.

$$\varphi^{\circ+} - \varphi^{\circ-} = \frac{\mathscr{E}\sigma R\alpha^* (1 - \delta\varphi)}{1 + \sigma R\alpha^*} \qquad (\delta\varphi = \delta\varphi^{\circ+}/\mathscr{E})$$

$$\eta^* = \frac{R\sigma\alpha^* (1 - \delta\varphi)}{1 + \sigma R\alpha^*} \qquad \left(\alpha^* = \frac{K(k')}{K(k)}, \quad k^2 + k'^2 = 1, \quad k = \operatorname{sech}\frac{\pi\lambda}{h}\right) \tag{4.1}$$

In (4.1), $K(k)$ is the complete elliptic integral of the first kind. The graph of the function α^* is given in [8]. When $2\lambda/h > 0.3$, we have [9 and 10]

$$\alpha^* = 2\lambda / h + 2 \ln 2 / \pi$$

By (4.1) the electrode potential drop decreases the output current and the efficiency by the factor $(1 - \delta\varphi)$.

It should be noted that if the electrodes had different lengths and were displaced with respect to each other, then the characteristics of the device would be given by formulas (4.1) in which α^* would have to be replaced by the quantity Φ defined by the formula (1.6) in [11]

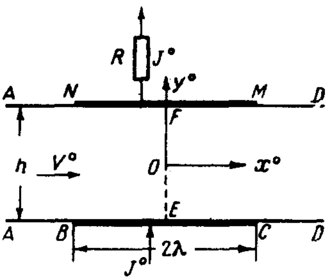

FIG. 2

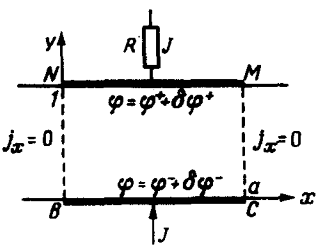

FIG. 3

5. Let us study the influence of the electrode potential drop on the effects caused by the non-uniformity of the magnetic field with respect to the length of the channel and transverse velocity gradient. Let us consider a channel of constant width h with electrodes BC and NM of length 2λ (fig. 3). Disregarding the assumption made in the derivation of the systems (2.4) and (2.5) we shall assume, that on the lines NB the condition $j_x = 0$ is fulfilled. This is possible if the conductivity in the electrode zone is regulated by the introduction, through the channel wall, of an easily ionised substance in the section NB while to the right of MC, the conductivity is equal to zero as a result of cooling of the fluid. The systems (2.4) and (2.5) are rewritten in the form

$$j_{x0} = -\frac{\partial\varphi_0}{\partial x}, \qquad j_{y0} = -\frac{\partial\varphi_0}{\partial y} + f, \qquad \triangle\varphi_0 = 0$$

$$j_{x0} = 0 \quad \text{on } NB,\ MC; \qquad \varphi = \varphi_0^+ \quad \text{on } NM; \qquad \varphi = \varphi_0^- \quad \text{on } BC \tag{5.1}$$

$$\varphi_0^+ - \varphi_0^- = R\sigma J_0 \qquad \left(J_0 = \int_0^a j_{y0}(x, 1)\, dx, \ a = \frac{2\lambda}{h}\right)$$

$$j_{x1} = -\frac{\partial\varphi_1}{\partial x}, \qquad j_{y1} = -\frac{\partial\varphi_1}{\partial y}, \quad \triangle\varphi_1 = 0 \tag{5.2}$$

$$j_{x1} = 0 \quad \text{on } NB,\ MC; \qquad \varphi_1 = \varphi_1^+ + s_0^+ \quad \text{on } NM; \qquad \varphi_1 = \varphi_1^- + s_0^- \quad \text{on } BC$$

$$\varphi_1^+ - \varphi_1^- = R\sigma J_1 \left(J_1 = \int_0^a j_{y1}(x, 1)\, dx, \ s_0^+ = s^+[j_{y0}(x, 1)], \ s_0^- = s^-[-j_{y0}(x, 0)]\right)$$

We note that the systems (5.1) and (5.2) also define the electric field in the transverse cross-section of the channel with the electrodes NM and BC and the nonconducting sides NB and MC, when $\mathbf{v}^\circ = (0, 0, -V^\circ(x^\circ))$, $\mathbf{B}^\circ = (-B^*, 0, 0)$. In this case the nonuniformity of the current is controlled by the decrease in the velocity near the side walls NB and MC,

Solution of the systems (5.1) and (5.2) is given by the formulas (the potential on the electrode BC is assumed to be equal to zero)

$$\varphi_0 = \varphi_0^+ y, \qquad j_{x0} = 0, \qquad j_{y0} = f - \varphi_0^+, \qquad J_0 = \frac{\zeta_1}{1+\sigma Ra}, \qquad \varphi_0^+ = \frac{\sigma R\zeta_1}{1+\sigma Ra} \tag{5.3}$$

$$\varphi_1 = \varphi_1^+ y + \frac{1}{2}[\psi_0(1-y) + \phi_0 y] + \sum_{n=1}^{\infty}\left[\psi_n \sinh\frac{n\pi(1-y)}{a} + \phi_n \sinh\frac{n\pi y}{a}\right] \times$$

$$\times \operatorname{cosech}\frac{n\pi}{a}\cos\frac{n\pi x}{a} \qquad \left(\zeta_1 = \int_0^a f(x)\,dx\right)$$

$$\phi_n = \frac{2}{a}\int_0^a s_0^+ \cos\frac{n\pi x}{a}\,dx, \quad \psi_n = \frac{2}{a}\int_0^a s_0^- \cos\frac{n\pi x}{a}\,dx \qquad (n = 0, 1, 2, \ldots)$$

$$\varphi_1^+ = -\frac{\sigma R\mu}{1+\sigma Ra}, \qquad J_1 = -\frac{\mu}{1+\sigma Ra} \qquad \left(\mu = \int_0^a (s_0^+ - s_0^-)\,dx\right) \tag{5.4}$$

The efficiency η^* when first two approximations are taken into account, is equal to

$$\eta^* = \eta_0^*(1 + \varepsilon q)$$

$$\eta_0^* = \frac{\sigma R\zeta_1^2}{(1+\sigma Ra)\zeta_3}, \qquad q = \frac{1}{\zeta_3}[\nu(1+\sigma Ra) - \sigma R\mu\zeta_1] - \frac{2\mu}{\zeta_1}$$

$$\zeta_3 = \zeta_2(1+\sigma Ra) - \sigma R\zeta_1^2, \qquad \zeta_2 = \int_0^a f^2\,dx, \qquad \nu = \int_0^a f(s_0^+ - s_0^-)\,dx \tag{5.5}$$

Let us consider some particular cases. Let $f \equiv 1$. Assuming that $\delta\varphi^+ = \text{const} = \varepsilon$, $\delta\varphi^- = 0$, we obtain

$$j_x = 0, \qquad j_y = \frac{1-\varepsilon}{1+\sigma Ra}, \qquad \eta^* = \frac{\sigma Ra(1-\varepsilon)}{1+\sigma Ra} \tag{5.6}$$

These formulas constructed according to the first two approximations, give the complete solution of the problem, since the approximations of higher order are identically equal to zero.

If $\delta\varphi^+ = \varepsilon j_y$, $\delta\varphi^- = 0$, then, constructing the solutions for all approximations we find that

$$j_x = 0, \quad j_{yk} = \frac{(-1)^k}{(1+\sigma Ra)^{k+1}}, \quad j_y = \sum_{k=0}^{\infty}\varepsilon^k j_{yk} = \frac{1}{(1+\varepsilon+\sigma Ra)}, \quad \eta^* = \frac{\sigma Ra}{1+\varepsilon+\sigma Ra} \tag{5.7}$$

Formulas (5.6) are valid when $\varepsilon < 1$. The series in (5.7) converge when $\varepsilon < 1 + \sigma Ra$, but the finite expressions in (5.7) for j_y and η^* are valid for any ε (it can easily be confirmed by solving the initial systems of equations directly).

The simplicity of the corresponding solutions is explained by the absence of the zones of reverse currents at the electrodes.

Let us now suppose that there are zones of reverse current at the electrodes ($f \not\equiv 1$). s_0^+ and s_0^- are given by the formulas

$$s_0^+ = \xi(j_{y0})\, j_{y0}, \qquad s_0^- = -\xi(-j_{y0})\, j_{y0} \qquad \left(\xi(x) = \begin{cases} 1 \text{ when } x > 0 \\ 0 \text{ when } x < 0 \end{cases}\right) \tag{5.8}$$

The relations (5.8) correspond to the conditions $\delta\varphi \sim j_n$ and $\delta\varphi \equiv 0$ on the segments of the electrode with $j_n > 0$ and $j_n < 0$ respectively (fig. 4). It is easy to see that $s_0^+ - s_0^- = j_{y0}$ is also true. The quantity q is now equal to

$$q = \frac{(\sigma R)^2 am - \zeta_2}{(1 + \sigma Ra)(\sigma Rm + \zeta_2)} \qquad (m = a\zeta_2 - \zeta_1^2) \tag{5.9}$$

The function $q(\sigma R)$ increases monotonely from $q(0) = -1$ to $q(\infty) = 1$ and vanishes, when $\sigma R = (\sigma R)_* = (\zeta_2 / am)^{1/2}$.

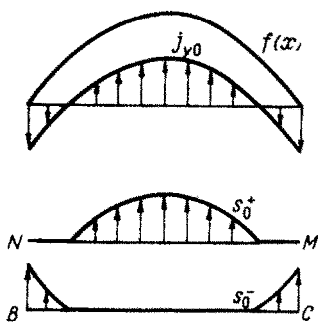

FIG. 4

We should note that, according to the Cauchy-Buniakovski inequality $m \geqslant 0$ where the equality sign applies only when $f \equiv$ const. For the same value of σR, the function $\eta_0^*(\sigma R)$ reaches a maximum (fig. 5).

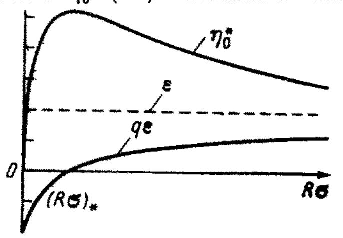

FIG. 5

Hence the electrode potential drop leads to a decrease in efficiency when $\sigma R < (\sigma R)_*$ (when the boundary effect is relatively weak), and to an increase when $\sigma R > (\sigma R)_*$ (when the boundary effect is significant). The current generated and the induced potential difference decrease for all σR. For large σR however, the sum of the Joule and electrode loses decreases strongly and this leads to the increase in efficiency $\eta^* \approx \eta_0^*(1 + \varepsilon)$. For small σR on the other hand $\eta^* \approx \eta_0^*(1 - \varepsilon)$.

Let us estimate the quantity $(\sigma R)_*$ in the problem of the distribution of current in the transverse cross-section of the channel. We shall choose

$$f(x) = \frac{\rho}{\rho\cosh\rho - \sinh\rho}\left[\cosh\rho - \cosh\frac{\rho(2x - a)}{a}\right] \tag{5.10}$$

to represent a family of velocity profiles [12], and we shall assume the mean velocity with respect to the cross-section to be the characteristic velocity. With changing the parameters ρ from 0 to ∞ the profile $f(x)$ is continuously deformed, passing from the Poiseuille profile to the complete one*. For the profiles (5.10)

* The family (5.10) represents the Hartmann profile.

$$\zeta_1 = a, \qquad \zeta_2 = \frac{a\rho\,(\rho + 2\rho\cosh^2\rho - 3/2\sinh 2\rho)}{2(\rho\cosh\rho - \sinh\rho)^2}$$

When $\rho \gg 1$ we have $(\sigma R)_* a \approx \sqrt{2\rho}$. Hence, with increasing fullness of velocity profile the region of variation of σR, in which the electrode potential drop decrease the efficiency, continuously increases.

6. Let us obtain the solutions of the systems (2.4) and (2.5) for the channel in fig. 2 in the case when $f \neq$ const, and consequently when the quantity j_n can change its sign at the electrodes. Let f be an even function. Then, as a result of symmetry, it is sufficient to consider just the right-hand half of the channel $x > 0$, assuming that $j_x = 0$ on the line *FOE*. The corresponding solution of the system (2.4) was obtained in [8]. The system (2.5) is rewritten in the form

$$j_{x1} = -\frac{\partial\varphi_1}{\partial x}, \qquad j_{y1} = -\frac{\partial\varphi_1}{\partial y}, \qquad \triangle\varphi_1 = 0$$

$$j_{y1} = 0 \quad \text{on} \quad CD,\ MD; \qquad j_{x1} = 0 \quad \text{on} \quad FOE$$

$$\varphi_1 = \varphi_1^+ + s_0^+ \quad \text{on} \quad FM, \qquad \varphi_1 = s_0^- \quad \text{on} \quad EC, \qquad \varphi_1^+ = 2\sigma R J_1 \tag{6.1}$$

$$\left(s_0^+ = s^+[j_{y0}(x, {}^1\!/_2)],\ s_0^- = s^-[-j_{y0}(x, -{}^1\!/_2)],\ J_1 = \int_0^{{}^1\!/_2 a} j_{y1}(x, -{}^1\!/_2)\,dx,\ a = 2\lambda/h\right)$$

Here it is assumed that the potential of the electrode BC is equal to zero. To solve the problem let us introduce the analytic function

$$w(z) = \frac{\partial\varphi_1}{\partial x} - i\,\frac{\partial\varphi_1}{\partial y} \qquad (z = x + iy)$$

According to the boundary conditions $\operatorname{Im} w = 0$ on the segment of the boundary *CDM* while on *MFEC*, $\operatorname{Re} w$ is known. Hence w can be found by means of the Keldysh-Sedov formula [13]. The solution has the form

$$w = \frac{1}{\pi g(t)} \int_k^1 \left[\left(\frac{1-\rho}{1+\rho}\right)^{1/2} \frac{\beta^-(\rho)}{\rho - t} - \left(\frac{1+\rho}{1-\rho}\right)^{1/2} \frac{\beta^+(\rho)}{\rho + t}\right] d\rho + \frac{\gamma}{\sqrt{(t-1)(t+1)}}$$

$$g(t) = \left(\frac{t-1}{t+1}\right)^{1/2}, \quad t = k\sin\pi i z, \quad \gamma = \frac{\pi r + 2\sigma R\,(i_1 - i_2 - i_3) - i_4}{2K(k)\,(1 + \sigma R\alpha^*)} \qquad \left(k = \operatorname{sech}\frac{\pi\lambda}{h}\right)$$

$$i_1 = \frac{1}{\pi}\int_k^1 \left(\frac{1+\tau}{(1-\tau)(\tau^2 - k^2)}\right)^{1/2} \left[\int_k^1 \left(\frac{1+\rho}{1-\rho}\right)^{1/2} \frac{\beta^+(\rho)\,d\rho}{\rho + \tau}\right] d\tau, \quad i_2 = \frac{1}{\pi}\int_k^1 \ln\frac{1-\tau}{\tau - k}\,\frac{\beta^-(\tau)\,d\tau}{\sqrt{\tau^2 - k^2}}$$

$$i_3 = \frac{1}{\pi}\int_k^1 \left(\frac{1+\tau}{(1-\tau)(\tau^2 - k^2)}\right)^{1/2} \left\{\int_k^1 \left[\left(\frac{1-\rho}{1+\rho}\right)^{1/2} \beta^-(\rho) - \left(\frac{1-\tau}{1+\tau}\right)^{1/2} \beta^-(\tau)\right] \frac{d\rho}{\rho - \tau}\right\} d\tau \tag{6.2}$$

$$i_4 = \frac{1}{\pi}\int_{-k}^{+k} \left(\frac{1+\tau}{(1-\tau)(k^2 - \tau^2)}\right)^{1/2} \left\{\int_k^1 \left[\left(\frac{1-\rho}{1+\rho}\right)^{1/2} \frac{\beta^-(\rho)}{\rho - \tau} - \left(\frac{1+\rho}{1-\rho}\right)^{1/2} \frac{\beta^+(\rho)}{\rho + \tau}\right] d\rho\right\} d\tau$$

$$\left(r = s_0^+ - s_0^- \text{ when } x = 0;\ \beta^+(\rho) = \frac{ds_0^+}{dx},\ \beta^-(\rho) = \frac{ds_0^-}{dx} \text{ when } x = x(\rho) = \frac{1}{\pi}\cosh^{-1}\frac{\rho}{k}\right)$$

In (6.2) the function $t(z)$ maps conformally the half strip $x > 0$, $|y| < {}^1\!/_2$ onto the upper half plane $\nu > 0$ (fig. 6), functions $K(k)$ and α^* are defined in (4.1), and the branches chosen for the square roots are those which are positive when $t = \tau > 1$. The

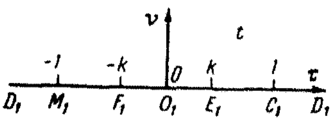

FIG. 6

potential difference φ_1^+ is given by the formula.

$$\varphi_1^+ = \frac{\sigma R\,[2(i_1 - i_2 - i_3) + \alpha^* i_4 - \pi\alpha^* r]}{\pi(1 + R\sigma\alpha^*)} \qquad (6.3)$$

If the current j_{y0} is positive along the length of the electrodes and it can be assumed that $s_0^- = 0$, $s_0^+ = \mathrm{const}$, then from (6.3) we find, that

$$\varphi_1^+ = -\frac{\sigma R\alpha^* s_0^+}{1 + \sigma R\alpha^*} \qquad (6.4)$$

7. Let us find the solution of the system (2.5) for the channel $|x| < \infty$, $0 < y < 1$ with infinite electrodes ND and BD (fig. 7). The corresponding solutions of the system (2.4) for various functions $f(x)$ were obtained in [14-16]. The boundary conditions (2.5) are written in the form

$$\frac{\partial\varphi_1}{\partial y} = 0 \quad \text{on } AB, AN; \qquad \varphi_1 = s_0^+ \quad \text{on } ND, \qquad \varphi_1 = s_0^- \quad \text{on } BD$$

$$(s_0^+ = s^+[j_{y0}(x, 1)],\; s_0^- = s^-[-j_{y0}(x, 0)]) \qquad (7.1)$$

In the problem under consideration the potential difference between the electrodes is assumed to be known. If the region under study is conformally mapped on the half-strip in the t-plane by means of the transformation $\exp(-\pi z) = \cos t$ (fig. 7), then the solution is easily constructed by Fourier's method

$$\varphi_1(\tau, \nu) = \frac{\varkappa_0}{2} + \sum_{n=1}^{\infty} \varkappa_n e^{-\nu n} \cos \tau n \qquad (t = \tau + i\nu)$$

$$\varkappa_n = \frac{2}{\pi}\int_0^{\pi} S(\tau)\cos n\tau\, d\tau, \qquad S(\tau) = \begin{cases} s_0^- \text{ when } 0 < \tau < \frac{1}{2}\pi, & \tau = \cos^{-1} e^{-\pi x} \\ s_0^+ \text{ when } \frac{1}{2}\pi < \tau < \pi, & \tau = \pi - \cos^{-1} e^{-\pi x} \end{cases} \qquad (7.2)$$

The currents J_1^- and J_1^+, flowing across the segments of the electrodes BE and NF will be equal to

$$J_1^- = \int_0^x j_{y1}(x, 0)\, dx = \sum_{n=1}^{\infty} \varkappa_n \sin \tau n, \qquad (\tau = \cos^{-1} e^{-\pi x})$$

$$J_1^+ = \int_0^x j_{y1}(x, 1)\, dx = \sum_{n=1}^{\infty} (-1)^{n+1} \varkappa_n \sin n\tau$$

If the magnetic field does not extend sufficiently far from the electrode zone, then some parts of the electrodes will have different directions of j_{y0}. Let us choose the following family of model profiles $j_{y0}(x)$ on the electrodes:

$$j_{y0}(x) = 1 - \frac{2\chi}{\pi}\sin^{-1} e^{-\pi x} \quad (1 \leqslant \chi < \infty) \qquad (7.3)$$

It is easy to see that $j_{y0}(0) = -(\chi - 1)$, $j_{y0}(\infty) = 1$; $j_{y0} < 0$ when $0 < x < x^*$, $j_{y0} > 0$ when $x^* < x < \infty$, $j_{y0}(x^*) = 0$, where $\pi x^* = -\ln \sin \pi/2\chi$. The segment of the electrode where $j_{y0} < 0$, becomes larger with the increase of χ

Consequently, the profiles (7.3) represent several of the characteristic peculiarities of the actual distribution of j_{y0} along the electrodes.

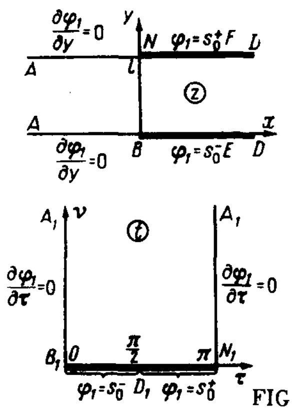

FIG. 7

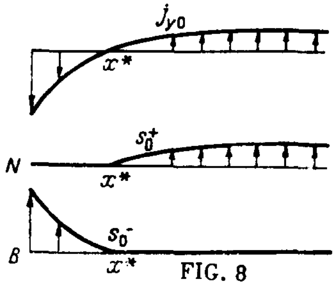

FIG. 8

Let us now suppose that the quantities s_0^+ and s_0^- are determined by the functions (5.8) in which j_{y0} is expressed by the formula (7.3) (fig. 8). In this case the coefficients are easily calculated and the currents J_1^- and J_1^+ take the form

$$J_1^- = \frac{4\chi}{\pi^2} I_1 + I_2 + I_3, \qquad J_1^+ = \frac{4\chi}{\pi^2} I_4 + I_2 - I_3$$

$$I_1 = \sum_{n=1}^{\infty} \frac{\sin n\tau}{n^2}, \qquad I_2 = -\frac{1}{\pi} \ln \tan\left(\frac{\pi}{4} + \frac{\tau}{2}\right) \qquad (\tau = \cos^{-1} e^{-\pi x})$$

$$I_3 = \frac{\chi}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n (1 - 2\cos n\pi/\chi)}{n^2} \sin 2n\tau, \qquad I_4 = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin n\tau}{n^2}$$

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